Frequency-controlled interaction between magnetic microspheres

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We show that the interaction between magnetic microspheres, fabricated by coating glass microspheres with a layer of nickel, can be controlled by varying the frequency of the applied magnetic field. By floating two such microspheres on the meniscus of glycerin and applying an ac magnetic field, it is shown that the spheres achieve an equilibrium separation owing to the balance between the repulsive dipole-dipole interaction and the "attractive" force due to the weight of the particles. A monotonic decrease of the magnetorheological effect with frequency increasing is observed. Good agreement between theory and experiment is observed. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2189830]

Magnetorheological (MR) suspensions are composed of micrometer-sized paramagnetic particles dispersed in a non-magnetic fluid; i.e., water or oil.[1,2] Under an external magnetic field, the particles interact through induced dipole moments, leading to rapid (on the order of millisecond) aggregation of the particles to form chain-like structures, with enhanced viscoelasticity and solid-like behavior as a result.[3] Such characteristics have a wide range of potential applications, ranging from active dampers, torque transducers, to robotics and vibration-control systems.[4] While most studies of MR effect to date have focused on rheological behavior under a dc magnetic field,[5,6] MR fluid's response to the rotating magnetic field, especially the dependence of MR fluid's structure and dynamics on the rotational frequency,[7] has recently been an area of interest. In this work, we present experimental and theoretical investigations on the frequency-dependent interaction between two magnetic microspheres. It is shown that the frequency tuning of the paramagnetic susceptibility can lead to frequency-controllable interaction between the paramagnetic microspheres, with increasing frequency implying decreasing interaction.

Paramagnetic particles are fabricated by coating uniformly sized glass microspheres [73(±2) μm in diameter] with ~8-μm-thick nickel layers, shown in the upper-right inset in Fig. 1. The nickel-coated microspheres are heated in vacuum at 400 °C for 2 h and then annealed at 550 °C for 3 h. The annealed microspheres possess a small magnetic moment of 10^{-6} emu.[8] This small magnetic moment will be ignored in our theoretical modeling of the effect because of the alternative magnetic field. Instead, the particles will be treated as paramagnetic.

Figure 1 demonstrates the experimental setup, where two spheres are placed on the surface of glycerin with an alternating magnetic field applied perpendicular to the fluid/air interface. The separation between the two microspheres is noted to change as a function of the field strength and frequency. The latter is seen from inset to Fig. 2. It is noted that with increasing frequency, the separation between the two microspheres decreases continuously, until touching occurs when the frequency is sufficiently high. We also observe from the experiments that at low frequencies the two microspheres vibrate slightly around the equilibrium position, following the external field. Such vibrations become more pronounced when the frequency was increased to 50 Hz, accompanied by decreasing separation between the two microspheres. With further frequency increase the vibration disappears, and the two microspheres approach each other until touching occurs at ~500 Hz (see inset to Fig. 2). The measured of frequency dependence distances are plotted in Fig. 2.

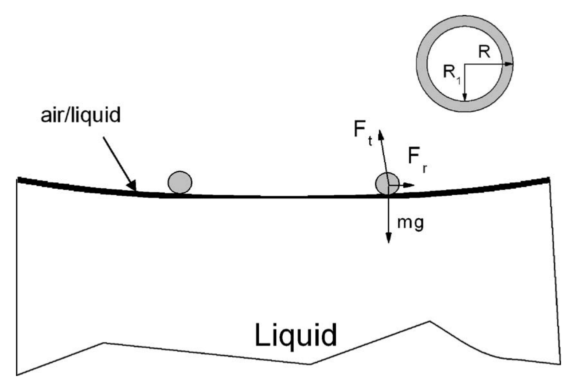

FIG. 1. The curved liquid meniscus, where two spheres are in the symmetric positions and the forces on one sphere are denoted. F_t is the surface tension, F_r is the repulsive force from dipole-dipole interaction, and mg is the gravity. A cross-sectional picture of the coated sphere is shown in the upper inset. R is the outer radius of the coating, and R_1 is the radius of the inner glass core.

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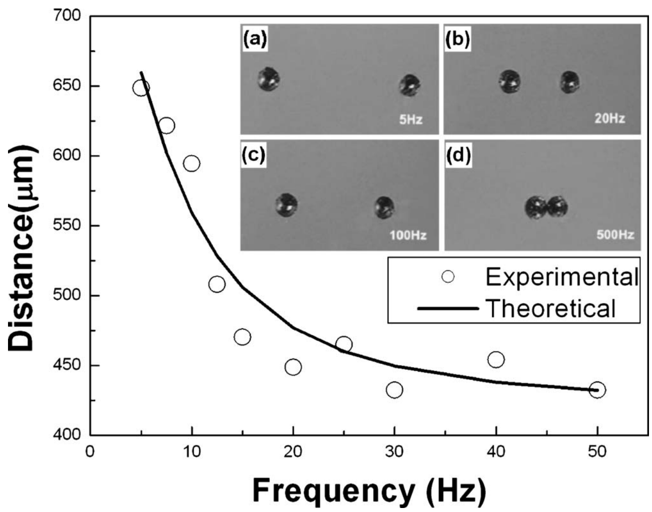

FIG. 2. Variation of distance r_{12} as a function of the field frequency, where the field strength is 15 G. Solid line is theory; circles are measured values. The motion of the two spheres is shown in the upper-right inset.

In order to explain our experimental observations, we have carried out theoretical analyses based on the dipole interaction model. We note that in glycerin, the surface tension force that a microsphere encounters is about 1.784 dyn, which is several orders of magnitude larger than the gravitational force: ~0.002 14 dyn. Hence, we can safely assume that the motion of the microspheres does not perturb the shape of the meniscus.

By using the Laplace formula with the appropriate boundary condition, the shape of the meniscus can be readily deduced as $z=z_0\{[I_0(\lambda r)-1]/[I_0(\lambda r_0)-1]\}$, where z denotes the surface height, with $z=0$ located at the center of the meniscus. $I_0(x)$ is the zeroth-order modified Bessel function of the first kind, $\lambda=\sqrt{\rho g/\sigma}$, with $\rho=1.26\times 10^3\ \mathrm{kg/m^3}$ the mass density of glycerin, g the gravitational acceleration, and $\sigma=63.4\ \mathrm{mJ/m^2}$ the surface tension of glycerin. Here the maximal value of $z=z_0$ and the radius of the container $r_0=0.5$ cm are given experimentally. The value of z_0 was measured photographically to be approximately 0.05 cm.

The induced magnetic moment is given by $\mathbf{u}=\tilde{\chi}\mathbf{h}$, where $\mathbf{h}$ is the external field, and $\tilde{\chi}$ is the complex magnetic susceptibility. The overhead tilda denotes the quantity to be complex; $\tilde{\chi}=\tilde{\mu}-1$, and $\tilde{\mu}$ is the complex magnetic permeability. Since Ni is a ferromagnetic material, hence its paramagnetic susceptibility can arise from domain boundary motion. Generally, when a ferromagnetic material is placed in an alternating magnetic field of small strength, its complex permeability can be expressed as $\tilde{\mu}=\mu_r-i\mu_i$, where $\mu_r=\mu_0+\mu_n/(1+\omega^2\tau^2)$ and $\mu_i=\mu_n\omega\tau/(1+\omega^2\tau^2)$. Here μ_0 is the frequency-independent permeability, and $\mu_n=\mu_n(h)$ is the magnitude of the strength-dependent component of the permeability, which is a function of the applied magnetic field strength. Here the paramagnetic susceptibility is due to the nickel coating. $\omega=2\pi f$ is the angular frequency, and τ denotes the relaxation time.

According to the expression for the magnetostatic energy,[8–10] the dipole-dipole interaction between two microspheres[11–13] can be expressed as $F_r=3\mu_m u^2/r_{12}^4$, where r_{12} is the distance between the two microspheres. By projecting the gravity and magnetic repulsive force along the surface tangent, the equilibrium relation can be derived as

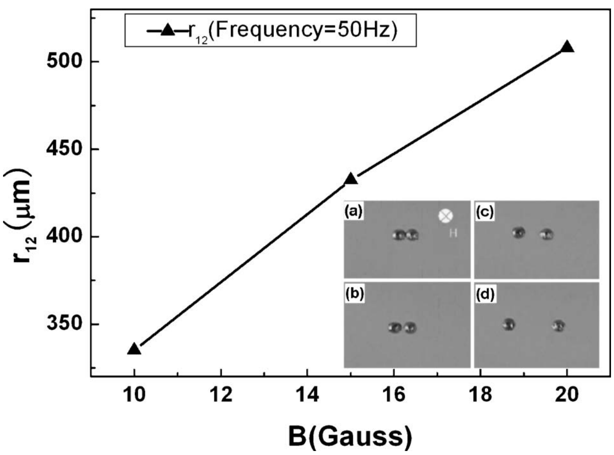

FIG. 3. Variation of the distance r_{12} between two spheres as a function of the field strength, where the field frequency is 50 Hz. The motion of the two spheres is shown in the lower-right inset.

$$\frac{3\mu_m(|\tilde{\chi}|h)^2}{r_{12}^4} = \frac{4}{3}\pi g[\rho_1 R_1^3 + \rho_2(R^3 - R_1^3)]\frac{z_0\lambda I_1(\lambda r_{12}/2)}{I_0(\lambda r_0) - 1}, \qquad (1)$$

where $\mu_m=1$ (we regard the glycerin and air to be the uniform medium since they bear similar magnetic parameters), the density of glass $\rho_1=2.7\text{–}3.5\times 10^3\ \mathrm{kg/m^3}$, the density of nickel $\rho_2=8.9\times 10^3\ \mathrm{kg/m^3}$; $I_1(x)$ is the first-order modified Bessel function of the first kind, and

$$|\tilde{\chi}|^2 = (\mu_0 - 1)^2 + \frac{2(\mu_0 - 1)\mu_n + \mu_n^2}{1 + (2\pi\tau)^2 f^2}. \qquad (2)$$

When the frequencies (5 to 50 Hz) and the field are small, we may assume τ and μ_0 are constant. $\mu_n(h)$ as well as τ and μ_0, can be obtained by fitting one set of experimental data at a fixed field. $\mu_n(h)$ under the other field strengths can be obtained directly from Eq. (1) through the experimental data for the separation of the two spheres at a fixed frequency, after τ and μ_0 are determined. The validity of these assumptions can be verified by predicting and comparing with further experimental data with the same parameters.

We fit the experimental data for the distance of the two spheres versus the frequency obtained under field strength of 15 G with Eq. (1), as shown in Fig. 2. The three fitting parameters are obtained: $\tau=3.1955\times 10^{-2}$, $\mu_0=1+3.935\,23\times 10^{-7}$, and $\mu_n=1.00941\times 10^{-6}$ (for $h=15$ G). These parameters are qualitatively in agreement with the previous work.[8,14]

With a fixed frequency, the distance between the two spheres increases when the field strength increases, as shown in the lower-right inset from (a) to (d) in Fig. 3. The curve in Fig. 3 shows the distance varying with the field strength. From the data for the distance, and the values for τ and μ_0 obtained earlier we can get from Eq. (1) that $\mu_n=1.168\,33\times 10^{-6}$ for $h=20$ G, and $\mu_n=0.744\,71\times 10^{-6}$ for $h=10$ G. The value μ_n for each field strength can be in turn used to predict the distance of two spheres varying with the frequency at this field strength, as shown in Fig. 4. As can be observed, the experimental data for the distance versus the frequency for two different field strengths show good agreement with the theory.

In summary, the frequency dependences of the interactions between two identically magnetic spheres floating on

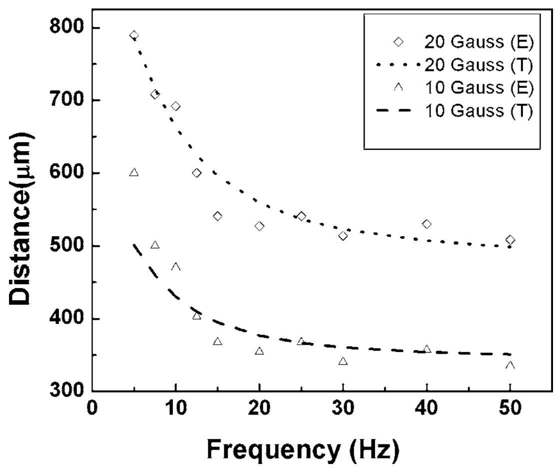

FIG. 4. Variations of distance as a function of the field frequency, for two different field strengths. The curves are theory and the symbols are experiment data.

the liquid meniscus under an alternating external magnetic field were investigated. It is found that such interactions decreased as the frequency of the magnetic field increased. At low frequency, two spheres separate from each other stably with a distance determined by the equilibrium between the repulsive force caused by the magnetic field induced dipole-dipole interaction of magnetic microspheres and the "attractive" force by the weight of particles projected along the surface tangent. The separation gets smaller with the frequency increasing, which indicates a degenerating interaction between the particles as the frequency increases. The experiment is in agreement with the theoretical calculation based on the dipole-dipole interaction. The results presented here may explain the degeneration of magnetorheological effect of MR suspension performed at high frequency.

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